

# A finite-energy solution in Yang-Mills theory and quantum fluctuations.

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## Abstract

A finite-energy solution of Yang-Mills theory with a nonstandard lagrangian is provided. Properties of these solution are studied and also a possible physical interpretation is given.

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## 1 Introduction.

This paper is devoted to constructing of the stable finite-energy and compact gluon object in the Yang-Mills theory with a nonstandard lagrangian. This lagrangian consists of pure Yang-Mills lagrangian  $\mathcal{L}_{YM} = -1/4 (F_{\mu\nu})^a (F^{\mu\nu})^a$  and higher derivative term  $\Delta\mathcal{L}$  related to quantum fluctuations of the gluon field. Such an approach to investigation of quantum fluctuations had been introduced in the series of papers in the middle of 80's [1]-[9]. In these papers IR low-energy limit of QCD have been studied and there was demonstrated that contribution from the quantum fluctuations can be taken into account by modification of QCD lagrangian via introducing into the lagrangian the additional higher derivative terms like  $\epsilon^{abc} (F_{\mu\nu})^a (F^\nu_\rho)^b (F^{\rho\mu})^c$  or  $(D_\rho F_{\mu\nu})^a (D^\rho F^{\mu\nu})^a$ . From methodological point of view lagrangians obtained within this approach are very similar to the well-known Euler-Heisenberg effective lagrangian in QED [10]. As a result, in leading approximation for gluon field one obtained the theory with a new lagrangian  $\mathcal{L} = \mathcal{L}_{YM} + \Delta\mathcal{L}$  for the  $c$ -number field  $A_{eff}$  and this classical Yang-Mills field is an average of the initial gluon field  $A_0$  over quantum fluctuations:  $A_{eff} = \langle A_0 \rangle$ . In [3, 4, 5] the investigation of classical solutions in such effective theories was discussed in context of the color confinement problem. A very similar problem is discussed in paper [11] also.

There are many approaches to obtaining finite-energy compact gluon objects from QCD. The crucial point here stems from the fact that pure classical Yang-Mills theory hasn't such solutions by reason of scale invariance. In the papers [12, 13] it was shown that typical spherically symmetrical solution of pure Yang-Mills theory is an infinite-energy solution with singularity on the finite radius sphere. Therefore, any approaches to finding gluon clusters is based on the

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various modifications of lagrangians. For example, the well-known monopole solution [14] exists thanks to the interaction of Yang-Mills field with the field of matter. Furthermore, some kind of gluon finite-energy objects appear in lattice approach to the QCD [15]. Such solutions are called glueballs. A glueball on the lattice is a quantum object having no analogues in classical field theory. Unfortunately such colorless gluon object wasn't be observed by experiment yet and only model predictions for physical characteristics like mass, effective radius and so on exist now.

In the present work we try to find similar quantum compact finite-energy objects by using of an effective low-energy approach to the QCD that was discussed above.

## 2 Finite-energy gluon clusters.

In this section we investigate the classical Yang-Mills theory with a nonstandard modified lagrangian

$$\mathcal{L}_{YM}^\varepsilon = -\frac{1}{4}(F_{\mu\nu})^a(F^{\mu\nu})^a - \frac{\varepsilon^2}{6}\epsilon^{abc}(F_{\mu\nu})^a(F^\nu{}_\rho)^b(F^{\rho\mu})^c \quad (1)$$

where  $\varepsilon = 1/M$  is an inverse mass dimensional parameter characterizing the intensity of quantum fluctuation;  $(F_{\mu\nu})^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc}A_\mu^b A_\nu^c$  and  $(D_\mu)^{ab} = \delta^{ab}\partial_\mu + \epsilon^{abc}A_\mu^c$ . Here we deal with  $SU(2)$  Yang-Mills field.

Such form of modification of the Yang-Mills lagrangian is chosen due to the fact that the theory obtained contains only second-order derivative terms. Thus the dynamics of this field theory can be studied in detail.

Using the variation principle, we get the equation of motion

$$D_\mu^{ab}(F^{\mu\nu} - \varepsilon^2 G^{\mu\nu})^b = 0. \quad (2)$$

where  $(G^{\mu\nu})^a = \epsilon^{abc}(F^\nu{}_\rho)^b(F^{\rho\mu})^c$ .

Adding the divergence

$$\partial_\rho[(F^{\nu\rho} - \varepsilon^2 G^{\nu\rho})^a \partial_\mu A_\rho^a], \quad (3)$$

to the energy-momentum tensor

$$T^\nu{}_\mu = \partial_\mu A_\rho^a \frac{\partial \mathcal{L}_{YM}^\varepsilon}{\partial(\partial_\nu A_\rho^a)} - \delta^\nu{}_\mu \mathcal{L}_{YM}^\varepsilon = -(F^{\nu\rho} - \varepsilon^2 G^{\nu\rho})^a \partial_\mu A_\rho^a - \delta^\nu{}_\mu \mathcal{L}_{YM}^\varepsilon, \quad (4)$$

we obtain the symmetrical form of this tensor

$$T^\nu{}_\mu = -(F^{\nu\rho} - \varepsilon G^{\nu\rho})^a (F_{\mu\rho})^a - \delta^\nu{}_\mu \mathcal{L}_{YM}^\varepsilon. \quad (5)$$

Now we consider the spherically symmetrical chromomagnetic field configuration. Substituting the well-known [16] Wu-Yang ansatz

$$A_0^a = 0, \quad A_i^a = \epsilon_{aij} n_j \frac{1 - H(r)}{r}, \quad n_i = x_i/r \quad r = \sqrt{x_i^2}, \quad (6)$$

in (2), we get the following equation on amplitude  $H(r)$

$$\begin{aligned} \left(1 - \frac{\varepsilon^2}{r^2}(H(r)^2 - 1)\right) r^2 H(r)'' &= H(r) (H(r)^2 - 1) + \\ &+ \frac{\varepsilon^2}{r^2} \left( (rH(r)')^2 H(r) - 2rH(r)'(H(r)^2 - 1) \right). \end{aligned} \quad (7)$$

Energy of field configuration generated by the solution of equation (7)  $H(r)$  is the functional

$$E^\varepsilon[H] = \int T^{00} d^3x = 4\pi \int_0^\infty \left[ \left( 1 - \frac{\varepsilon^2}{r^2} (H(r)^2 - 1) \right) (H(r)')^2 + \frac{(H(r)^2 - 1)^2}{2r^2} \right] dr = \int_0^\infty E(r) dr. \quad (8)$$

The next aim of our investigation is finding the solutions of equation (7). Notice that only finite-energy solutions are interesting for us. Hence the functional  $E^\varepsilon[H]$  (8) should be finite on such solutions.

Equation (7) is a very complicated nonlinear differential equation. In order to solve it only numerical or approximation methods seem applicable. The crucial point of such analysis is that the leading derivative term in this equation contains the factor

$$\Phi[H](r) = (r^2 - \varepsilon^2(H(r)^2 - 1)). \quad (9)$$

If  $H_s(r)$  is a solution of equation (7) and there is a point  $r = R$  such that  $\Phi[H_s](R) = 0$ , then this solution  $H_s$  has singular behavior in a neighborhood of this point  $r = R$  due to smallness of the factor  $\Phi[H_s]$ . Using the standard procedure, one obtains the asymptotic behavior near this point

$$H_s(r) \xrightarrow{r \rightarrow R \pm 0} \pm \sqrt{1 + R^2/\varepsilon^2} - C(R - r)^{2/3} + \underline{Q}(R - r), \quad (10)$$

where  $C$  is a constant. Of course, the  $H_s(R)$  finite but its derivative at the point  $r = R$  is singular. Indeed,

$$H'_s(r) \xrightarrow{r \rightarrow R \pm 0} \frac{2}{3} C (R - r)^{-1/3} + \underline{Q}(1) \rightarrow \infty. \quad (11)$$

Such singular behavior is analogous to the singular behavior on finite sphere of solutions of pure Yang-Mills field [13] discussed above but there is a principal difference. Energy of such solutions in the pure Yang-Mills case is infinite but in the modified Yang-Mills case energy (and other physical characteristics) of solution with singular behavior (11) should be *finite*:

$$E(r)|_{r=R} \sim 4\pi \left( \pm \frac{8\varepsilon^2}{9R^2} \sqrt{1 + R^2/\varepsilon^2} C^3 + \frac{R^2}{2\varepsilon^4} \right) < \infty. \quad (12)$$

Therefore such solutions are *physical*.

Now we should discuss the numerical investigation of solutions of equation (7) that have the asymptotic behavior (11) at some point  $r = R$ .

To guarantee stability of our solutions we should choose this asymptotics at origin ( $r \rightarrow 0$ )

$$H(r) \simeq -1 + a_1 r^2 + a_1^2 \frac{2\varepsilon^2 a_1 - 3}{10(1 + 2a_1 \varepsilon^2)} r^4 + \underline{Q}(r^6), \quad (13)$$

and at infinity ( $r \rightarrow \infty$ )

$$\begin{aligned} H(\rho) \simeq 1 + a_2 \rho + \frac{3}{4} a_2^2 \rho^2 + \frac{11}{20} a_2^3 \rho^3 + \frac{193a_2^2 - 240\varepsilon^2}{480} a_2^2 \rho^4 + \\ + \frac{329a_2^2 - 1280\varepsilon^2}{1120} a_2^3 \rho^5 + \underline{Q}(\rho^6), \quad \rho = 1/r, \end{aligned} \quad (14)$$

where  $a_1 > 0$  and  $a_2 > 0$  are constants. Solutions with such asymptotic are stable because vacuum states  $H(0) = -1$  and  $H(\infty) = 1$  are different.

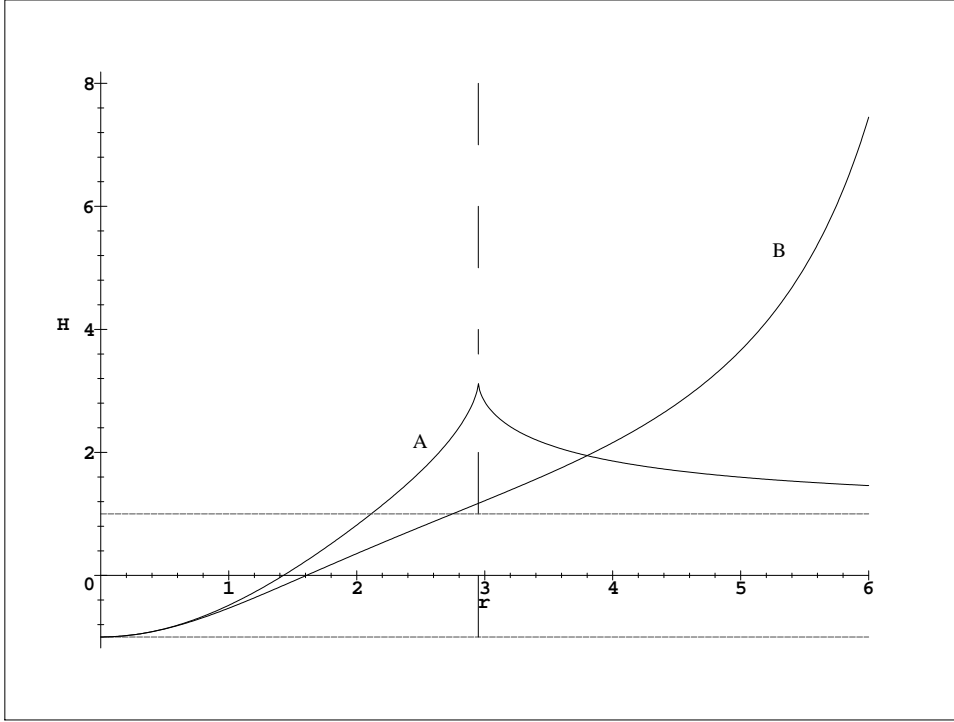


Figure 1: Gluon cluster object. Amplitude  $H(r)$  ( $\varepsilon = 1$ ,  $a_1 = 0.544$  and  $R = 1.425$ ).

Notice that equation (7) has very useful symmetries. First of all, this equation is symmetrical with respect to the changes  $H \leftrightarrow -H$ . So, if we have a solution  $H(r)$ , then  $-H(r)$  is a solution too.

Now let  $\varepsilon = \varepsilon_1$  and we have a set of solutions  $\{H_{\varepsilon_1}(r)\}$ . If we perform the change of variable

$$r \rightarrow \frac{\varepsilon_1}{\varepsilon_2}r, \quad \{H_{\varepsilon_1}(r)\} \xrightarrow{r \rightarrow \varepsilon_1 r / \varepsilon_2} \{H_{\varepsilon_2}(r) = H_{\varepsilon_1}\left(\frac{\varepsilon_1}{\varepsilon_2}r\right)\}, \quad (15)$$

we get equation (7) again but with new  $\varepsilon = \varepsilon_2$ . If we know a solution for some  $\varepsilon > 0$ , say,  $\varepsilon = 1$ , then using (15) we can obtain a solution for any other  $\varepsilon_1 > 0$ .

The numerical investigation of equation (7) is presented in Fig.1. The solution of pure Yang-Mills theory ( $\varepsilon = 0$ ) is shown by the line B. The function A is the solution of equation (7) with  $\varepsilon = 1$ . In Fig.2 we can see the energy density (8) corresponding to this solution.

The solution  $H_<$  starting at the origin (or *internal*) increases monotonically and its energy density increases too. Evidently, as energy density grows, the role of quantum fluctuations grows too. At the point  $r = R$ , the energy density attains its critical value  $E^{cr}$  and  $H(r)$  becomes singular (10). The solution  $H_>$  starting at the infinity (or *external*) demonstrates an absolutely similar behavior. Now, a very essential question arises: How to connect these two sets of solutions and how to determine such solutions on the whole space?

These questions have no mathematical answer because in this case we deal with the solutions that can not be extended to the right (to the left) because the point of singularity  $r = R$  is essential.

Obviously, this nonuniqueness of solution in the whole space is due to underdetermination of our effective model. It is necessary to introduce some additional physical condition that would allow to choose a physically reasonable solution from the broad class of solutions described above.

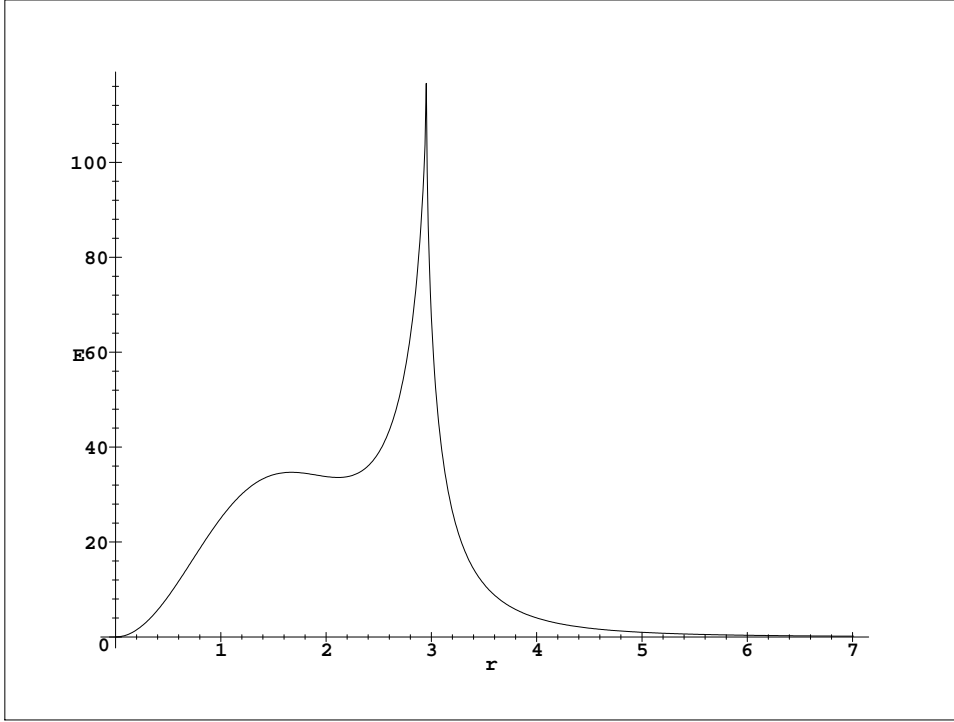


Figure 2: Gluon cluster object. Density of energy.

Since this solution of the model (1) has to be an effective approximation to an existing gluon object, the general properties of the latter should be represented by the former. Thus, if energy density of this gluon object is continuous everywhere, then it should be continuous for the approximating solution of equation (7) as well. We show below that the condition of continuity of energy density is sufficient for the construction of a unique solution and investigation of its properties.

According to mathematical structure of solutions of this model, the condition of continuity of energy density can be formulated as follows: *There exists a critical density of the energy for classical solutions in our effective Yang-Mills theory and the value of this critical density  $E^{cr}$  is a physical property of the theory. Therefore  $E^{cr}$  shouldn't depend on kind of solution (internal or external).* It follows that

$$E(r)_{<|_{r=R}} = E(r)_{>|_{r=R}} \implies C_{<} = C_{>}. \quad (16)$$

It is easily shown that condition (16) uniquely determines our solution and its properties ( $a_1$ ,  $a_2$  and  $R$ ) for any  $\varepsilon$ . This solution is shown in Fig.1 (function A). This solution looks like a shell with radius  $R$ .

Using (15), one obtains the following expression for energy of such gluon cluster

$$E^\varepsilon = \frac{1}{\varepsilon} E^{\varepsilon=1} = M E^{\varepsilon=1}, \quad (17)$$

where  $E^{\varepsilon=1} = 110.75$  is the energy of field configuration if  $\varepsilon = 1$ . Expression (17) is intuitively clear. Indeed, the pure Yang-Mills theory is scale invariant and has no mass-dimensional parameters. Modified Yang-Mills theory (1) has such parameter  $\varepsilon = 1/M$  and the mass of gluon objects under investigation is proportional to this parameter.

Now to predict the physical mass  $M_{cluster}$  and effective radius we should have a prediction of the value of parameter  $\varepsilon = 1/M$ . In this paper, following [4], we proposed that  $M \simeq 0.59\pi GeV$ , and our model gives the following prediction of the mass and effective radius of investigated gluon clusters:

$$M = 1/\varepsilon \simeq 0.59\pi GeV, \quad M_{cluster} \simeq 205 GeV, \quad R \simeq 0.15 fm \quad (18)$$

In the next section we give some conclusions and perspectives of such investigations are discussed.

### 3 Conclusions.

The aim of this paper is to show that quantum fluctuations of nonabelian Yang-Mills field can lead to generation of the stable cluster finite-energy solution. In our work we used the gauge-invariant approach [1, 2, 3, 8, 9] in which such quantum fluctuation should be taken into account by adding high-derivative terms to the pure Yang-Mills lagrangian. In the present work, we investigated the effective  $SU(2)$  Yang-Mills theory and chromomagnetic spherically symmetrical field configurations.

One of the interesting consequences of this effective theory is a fact that for the investigated field configuration there exists a critical value of energy density. This fact is due to the physical condition of continuity of energy density. Such condition allowed us to construct the cluster solution for all space points. We predicted that the mass of such object should be about two hundred GeV and effective radius should be about 0.2 fm.

Of course, we do not give a comprehensive investigation of this effective Yang-Mills theory. The questions about dyon solution or about a role of contributions from other high derivative modified term in pure Yang-Mills lagrangian are clear now. But maybe the most important question in such investigation is about physical consequences of existence of such gluon cluster objects and about their experimental status. All of these questions should be the themes for future investigation.

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### References

- [1] A.I. Alekseev, B.A. Arbuzov, V.A. Baikov, Theor.Math.Phys. 52 (1982) 739.
- [2] A.I. Alekseev, B.A. Arbuzov, Theor.Math.Phys. 59 (1984) 372.
- [3] A.I. Alekseev, B.A. Arbuzov, Phys.Lett. B 242 (1990) 103.
- [4] A.I. Alekseev, B.A. Arbuzov, Theor.Math.Phys. 65 (1986) 1106.
- [5] A.I. Alekseev, A.S. Vshivtsev, A.V. Tatarintsev 77 (1989) 1189.
- [6] A.I. Alekseev, A.S. Vshivtsev, V.K. Peres- Fernandes, Sov.Phys.J. 33 (1990) 352.

- [7] A.G. Lavkin, Sov.J.Nucl.Phys. 55 (1992) 644.
- [8] M. Baker, F. Zachariasen, Phys.Lett. B 108 (1992) 206.
- [9] M. Baker, J.S. Ball, F. Zachariasen, Nucl.Phys. B 209 (1983) 445.
- [10] E. Euler, Ann. Phys. 26 (1936) p.398; E. Euler, W. Heisenberg, Z. Phys. 98 (1936) p.714.
- [11] S. Baskal, T. Dereli, J.Phys. G 19 (1993) 477.
- [12] F.A.Lunev Phys. Lett. B 311 (1993) 273.
- [13] F.A. Lunev, O.V. Pavlovsky, Theor.Math.Phys. 117 (1998) 1241, hep-ph/9609452.
- [14] A. M. Polyakov, JETP Lett. 20 (1974) 194; G. 't Hooft, Nucl.Phys. B79 (1974) 276.
- [15] H. Fritzsch, M. Gell-Mann, in: Proc. of the XVI International Conference on High Energy Physics, Vol. 2, Chicago, 1972, p.135; G.B. West, Phys. Rev. Lett. 77 (1996) 2622; G.S. Bali et al., UKQCD Collaboration, Phys. Lett. B 309 (1993) 378; H. Chen, J. Sexton, A. Vaccarino, D. Weingarten, Nucl. Phys. B (Proc. Suppl.) 34 (1994) 357; F.E. Close, G.R. Farrar, Zh. Li, Phys. Rev. D55 (1997) 5749.
- [16] T.T. Wu, C.N. Yang, in: H.Mark, S.Fernback (Eds.), Properties of matter under unusual condition, N.Y., Interscience, 1969.